2.1 The Algebra of Spacetime

In spacetime we have 4 linearly independent vectors e_μ, $\mu = 0, 1, 2, 3$. Let us consider *flat* spacetime. It is convenient then to take orthonormal basis vectors γ_μ

$$\gamma_\mu \cdot \gamma_\nu = \eta_{\mu\nu} \tag{1}$$

where $\eta_{\mu\nu}$ is the diagonal metric tensor with signature $(+ - - -)$.

Clifford algebra in V_4 is called the *Dirac algebra*. Writing $\gamma_{\mu\nu} \equiv \gamma_\mu \wedge \gamma_\nu$ for a basis bivector, $\gamma_{\mu\nu\rho} \equiv \gamma_\mu \wedge \gamma_\nu \wedge \gamma_\rho$ for a basis trivector and $\gamma_{\mu\nu\rho\sigma} \equiv \gamma_\mu \wedge \gamma_\nu \wedge \gamma_\rho \wedge \gamma_\sigma$ for a basis quadrivector we can express an arbitrary number of Dirac algebra as

$$D = \sum_r D_r = d + d^\mu \gamma_\mu + \frac{1}{2!} d^{\mu\nu} \gamma_{\mu\nu} + \frac{1}{3!} d^{\mu\nu\rho} \gamma_{\mu\nu\rho} + \frac{1}{4!} d^{\mu\nu\rho\sigma} \gamma_{\mu\nu\rho\sigma} \tag{2}$$

where d, d^μ, $d^{\mu\nu}, \ldots$ are scalar coefficients.

Let us introduce

$$\gamma_5 \equiv \gamma_0 \wedge \gamma_1 \wedge \gamma_2 \wedge \gamma_3 = \gamma_0 \gamma_1 \gamma_2 \gamma_3 \ , \qquad \gamma_5^2 = -1 \tag{3}$$

which is the unit element of 4-dimensional volume and is called *pseudoscalar*. Using the relations

$$\gamma_{\mu\nu\rho\sigma} = \gamma_5 \epsilon_{\mu\nu\rho\sigma} \tag{4}$$

$$\gamma_{\mu\nu\rho} = \gamma_{\mu\nu\rho\sigma} \gamma^\rho \tag{5}$$

where $\epsilon_{\mu\nu\rho\sigma}$ is the totally antisymmetric tensor and introducing the new coefficients

$$S \equiv d \ , \quad V^\mu \equiv d^\mu \ , \quad T^{\mu\nu} \equiv \frac{1}{2} d^{\mu\nu}$$

$$C_\sigma \equiv \frac{1}{3!} d^{\mu\nu\rho} \epsilon_{\mu\nu\rho\sigma} \ , \qquad P \equiv \frac{1}{4!} d^{\mu\nu\rho\sigma} \epsilon_{\mu\nu\rho\sigma} \tag{6}$$

we can rewrite D of eq.(2) as the sum of scalar, vector, bivector, pseudovector and pseudoscalar part:

$$D = S + V^\mu + T^{\mu\nu} \gamma_{\mu\nu} + C^\mu \gamma_5 \gamma_\mu + P \gamma_5 \tag{7}$$

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