Behavior of spinors under Lorentz transformations

Rotated object

$a = 0,1,2,3$

Let us consider the following transformation of the basis vectors

(5) $$\gamma_a \rightarrow \gamma'_a = \mathrm{R}\,\gamma_a \mathrm{R}^{-1}$$

R is a proper or improper Lorentz transformation

A generalized spinor, $\Phi \in Cl(1,3)$, composed of γ_a, then transforms according to

$$\Phi = \psi^{\tilde{A}} \xi_{\tilde{A}} \rightarrow \Phi' = \psi^{\tilde{A}} \xi'_{\tilde{A}} = \psi^{A} \mathrm{R}\, \xi_B \mathrm{R}^{-1} = \mathrm{R}\,\Phi\,\mathrm{R}^{-1}$$

The transformation (2) of the basis vectors has for a consequence that the object Φ does not transform only from the right, but also from the left.

Piazzese 1993: Spinors cannot be interpreted as the minimal ideals of Clifford algebras

But: If the reference frame transforms as

$$\gamma_a \rightarrow \gamma'_a = \mathrm{R}\,\gamma_a$$

then

$$\Phi = \psi^{\tilde{A}} \xi_{\tilde{A}} \rightarrow \Phi' = \psi^{\tilde{A}} \xi'_{\tilde{A}} = \psi^{A} \mathrm{R}\, \xi_B = \mathrm{R}\,\Phi$$

Transformation of a spinor

The ideal approach is OK