Clifford algebra and spinors in Minkowski space

$\gamma_a = (\gamma_0, \gamma_1, \gamma_2, \gamma_3)$

Witt basis

$$\theta_1 = \frac{1}{2}(\gamma_0 + \gamma_3), \quad \theta_2 = \frac{1}{2}(\gamma_1 + i\gamma_2),$$
$$\bar{\theta}_1 = \frac{1}{2}(\gamma_0 - \gamma_3), \quad \bar{\theta}_2 = \frac{1}{2}(\gamma_1 - i\gamma_2)$$

The new basis vectors satisfy

$$\{\theta_a, \bar{\theta}_b\} = \eta_{ab}, \quad \{\theta_a, \theta_b\} = 0, \quad \{\bar{\theta}_a, \bar{\theta}_b\} = 0$$

Fermionic anticommutation relations

We now observe that the product

$$f = \bar{\theta}_1 \bar{\theta}_2$$

satisfies

$$\bar{\theta}_a f = 0, \qquad a = 1, 2$$

f can be interpreted as `vacuum', and $\bar{\theta}_a$ can be interpreted as operators that annihilate f.

An object constructed as a superposition

$$\Psi = (\psi^0 \underline{1} + \psi^1 \theta_1 + \psi^2 \theta_2 + \psi^{12} \theta_1 \theta_2) f$$

is a 4-component spinor.