It is convenient to change the notation:

$$\Psi = (\psi^1 \underline{1} + \psi^2 \theta_1\theta_2 + \psi^3 \theta_1 + \psi^4 \theta_2) f = \psi^\alpha \xi_\alpha \,, \qquad \alpha = 1,2,3,4$$

Spinor basis

$$f = \bar\theta_1 \bar\theta_2$$

Even part $\Psi_L = (\psi^1 \underline{1} + \psi^2 \theta_1\theta_2)\bar\theta_1\bar\theta_2$

Odd part $\Psi_R = (\psi^3 \theta_1 + \psi^4 \theta_2)\bar\theta_1\bar\theta_2$

$$i\gamma_5 \Psi_L = -\Psi_L$$

$$i\gamma_5 \Psi_R = \Psi_R$$

$$\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$$

Under the transformations

$$\Psi \rightarrow \Psi' = \mathrm{R}\Psi \,, \qquad \mathrm{R} = \exp[\tfrac{1}{2}\gamma_{a_1}\gamma_{a_2}\varphi]$$

Ψ transforms as a Dirac spinor.

Example:
$$\mathrm{R} = \mathrm{e}^{\frac{1}{2}\gamma_1\gamma_2\varphi} = \cos\frac{\varphi}{2} + \gamma_1\gamma_2 \sin\frac{\varphi}{2}$$

$$\Psi \rightarrow \Psi' = \mathrm{R}\Psi = \left(\mathrm{e}^{\frac{i\varphi}{2}} \psi^1 \underline{1} + \mathrm{e}^{-\frac{i\varphi}{2}} \psi^2 \theta_1\theta_2 + \mathrm{e}^{\frac{i\varphi}{2}} \psi^3 \theta_1 + \mathrm{e}^{-\frac{i\varphi}{2}} \psi^4 \theta_2 \right) f$$

This is the well-known transformation of a 4-component spinor.