## Four independent spinors

Four different possible vacua:

$$f_1 = \overline{\theta}_1 \overline{\theta}_2 \,, \qquad f_2 = \theta_1 \theta_2 \,, \qquad f_3 = \theta_1 \overline{\theta}_2 \,, \qquad f_3 = \overline{\theta}_1 \theta_2$$

Four different kinds of spinors:

$$\Psi^1 = (\psi^{11} \underline{1} + \psi^{21} \theta_1 \theta_2 + \psi^{31} \theta_1 + \psi^{41} \theta_2) f_1$$

$$\Psi^2 = (\psi^{12} \underline{1} + \psi^{22} \overline{\theta}_1 \overline{\theta}_2 + \psi^{32} \overline{\theta}_1 + \psi^{42} \overline{\theta}_2) f_2$$

$$\Psi^3 = (\psi^{13} \overline{\theta}_1 + \psi^{23} \theta_2 + \psi^{33} \underline{1} + \psi^{43} \overline{\theta}_1 \theta_2) f_3$$

$$\Psi^4 = (\psi^{14} \theta_1 + \psi^{24} \overline{\theta}_2 + \psi^{34} \underline{1} + \psi^{44} \theta_1 \overline{\theta}_2) f_4$$

Each of those spinors lives in a different minimal left ideal of $Cl(1,3)$.

In general, complexified version

An arbitrary element of $Cl(1,3)$ is the sum:

$$\Phi = \Psi^1 + \Psi^2 + \Psi^3 + \Psi^4 = \psi^{\alpha i} \xi_{\alpha i} \equiv \psi^{\tilde{A}} \xi_{\tilde{A}}$$

$\alpha = 1,2,3,4; \quad i = 1,2,3,4$

$$\xi_{\tilde{A}} \equiv \xi_{\alpha i} = \{\underline{1} f_1,\ \theta_1 \theta_2 f_1, \ldots, \theta_1 f_4,\ \overline{\theta}_2 f_4,\ \underline{1} f_4,\ \overline{\theta}_1 \theta_2 f_4\},$$

Spinor basis of $Cl(1,3)$

Matrix notation:

$$\psi^{\alpha i} = \begin{pmatrix} \psi^{11} & \psi^{12} & \psi^{13} & \psi^{14} \\ \psi^{21} & \psi^{22} & \psi^{23} & \psi^{24} \\ \psi^{31} & \psi^{32} & \psi^{33} & \psi^{34} \\ \psi^{41} & \psi^{42} & \psi^{43} & \psi^{44} \end{pmatrix}, \qquad \xi_{\tilde{A}} \equiv \xi_{\alpha i} = \begin{pmatrix} f_1 & f_2 & \overline{\theta}_1 f_3 & \theta_1 f_4 \\ \theta_1 \theta_2 f_1 & \overline{\theta}_1 \overline{\theta}_2 f_2 & \theta_2 f_3 & \overline{\theta}_2 f_4 \\ \theta_1 f_1 & \overline{\theta}_1 f_2 & f_3 & f_4 \\ \theta_2 f_1 & \overline{\theta}_2 f_2 & \overline{\theta}_1 \theta_2 f_3 & \theta_1 \overline{\theta}_2 f_4 \end{pmatrix}$$

Spinor of the 2nd left ideal

Basis of the 2nd left ideal